

# ANALYSIS OF OPTIMAL STOCK PORTFOLIO INVESTMENT IN LQ45 INDEX USES THE MARKOWITZ MODEL AND SINGLE INDEX MODEL

Fadlan Alkindi<sup>1</sup>, Isfenti Sadalia<sup>2</sup>, Iskandar Muda<sup>3</sup>

<sup>1,2,3</sup>Master of Management, Postgraduate School, Universitas Sumatra Utara, Medan Corresponding Author: <sup>1)</sup>fadlanalkindi@gmail.com,<sup>2)</sup>isfenti@usu.ac.id

#### Abstract

Based on the results of the optimal portfolio analysis of the Markowitz model and Single Index Model, it can be concluded as follows: Based on the optimal portfolio formation of the Markowitz Model, 4 shares form a portfolio return expectation of 0.0074, while the portfolio risk is 0.0428 and the proportion of funds formed is BBCA 50.81%, EXCL 9.83%, ICBP 30.59%, and KLBF 8.77%. Based on the formation of a single index model portfolio, 2 optimal portfolio formations were obtained with a portfolio return of 0.1486 and a risk of 0.0873, while the proportion of funds formed by ANTM was 10.5%, and BBCA was 89.5%.. Based on research results, it proves that the single index model can produce a profit of 14.86% with a risk level of 8.73% compared to the Risk-Free Asset Return Rate of 5.17%. Meanwhile, the Markowitz model can produce a portfolio return of 0.74% with a portfolio risk of 4.28%, which does not provide optimal profits because the expected return from the Markowitz model portfolio is lower than the Risk-Free Asset Return Rate.Lack of significant planning in investing by a company. This because in planning an investment project of course requires substantial funds, so if not budgeted and calculated properly, it can result in investment failure projects that can cause a company to experience large losses. This study discusses capital budgeting of a project in CV. ABC will buy a new machine. In the This study discussed how to calculate the initial investment, estimate the income that the company will get during the project, how long is the capital issued by the company for investment projects will be returned, and at most what is important is whether it is feasible or not is the investment project planning. Method used in capital budgeting calculations is the payback period, discounted payback period, Net Present Value (NPV), and Internal rate of Return (IRR). In the The results showed that CV ABC accepted the plan to purchase a corn drying machine by calculating the payback period for 5 years, the NPV and IRR are considered feasible.

Keywords: stock portfolio investment, LQ45, markowitz model, single index model

#### **INTRODUCTION**

Investment is a form of managing funds by placing these funds in an allocation that is expected to provide additional profits in the future. Investment is currently the main choice for people who want to increase the value of their wealth. There are two types of markets as a place to invest, namely the money market and the capital market. One of the instruments for investing in the capital market is in the form of shares. According to data quoted from the Indonesian Stock Exchange (BEI), investors who invest in shares always experience an increase every year. It is known that the number of shares traded from 2017 to 2019 has increased. The average number of shares traded in 2019 reached 7 trillion shares, an increase of around 40% from 2017 which reached 5 trillion. The increase in the number of shares traded indicates that investors feel safe investing in shares. Meanwhile, the number of issuers registered on the Indonesian Stock Exchange experienced a trend that continued to increase during 2015-2019. At the end of 2019, the number of issuers reached 658 issuers or an increase of 137 issuers from 2015. The presence of new issuers indicates that the stock market is still an important alternative for the business world to access cheap funds and support business expansion. Investing in the stock market can provide investors with the opportunity to obtain a higher rate of return than investing in the money market. However, the higher the rate of return received, the higher the risk faced. Therefore, investing contains a fairly high element of uncertainty. Every investor wants a rate of return that can be optimized from investment activities, with risks that can be minimized, and always looks for ways to obtain higher profits than the costs that must be borne by the investor (Larasati 2013).



Risk is divided into two, namely systematic risk and unsystematic risk. Widoatmodjo (2009) explains that systematic risk is a risk whose impact is felt by all investment instruments, such as inflation, rising interest rates, economic recession, and so on, while unsystematic risk is a risk whose impact is only felt by certain companies related to that risk. There are several types of portfolios, one of which is a stock portfolio. A stock portfolio is a collection of investment assets in the form of shares, both individual shares and company shares. Shares registered with PT. The Indonesian Stock Exchange is classified into 10 sectors according to industry classification. These sectors include the Agricultural Sector, Mining Sector, Basic Industry and Chemical Sector, Miscellaneous Industry Sector, Consumer Goods Industry Sector, Property, Real Estate and Building Construction Sector, Infrastructure, Utilities and Transportation Sector, Financial Sector, Trade, Services Sector and Investment, and Manufacturing Sector. The following is the development of 10 Sectoral Shares for the 2014-2019 period in Table 1.

Inductory			Year			
Industry	2014	2015	2016	2017	2018	2019
Agriculture	9.86%	- 26.87%	8.43%	-13.3%	-3.21%	-2.55%
Mining	-4.22%	- 40.75%	70.73 %	15.11 %	11.45%	- 12.83%
Basic Industry and Chemicals	13.09 %	- 24.98%	31.96 %	28.06 %	24.01%	14.44%
Miscellaneous Industry	8.47%	- 19.11%	29.64 %	0.77%	0.96%	- 12.23%
Consumer Goods Industry	22.21 %	-5.19%	12.56 %	23.11 %	- 10.21%	- 20.11%
Property, Real Estate and Building Construction	55.76 %	-6.47%	5.47%	-4.31%	-9.64%	12.54%
Infrastructure, Utilities and Transportation	24.71 %	- 15.42%	8%	12.14 %	- 10.09%	6.88%
Finance	35.41 %	-6.10%	18.17 %	40.52 %	3.05%	15.22%
Trade, Service and Investment	13.11 %	-3.31%	1.31%	7.08%	- 14.94%	-1.79%
Manufacturing	16.04 %	- 13.75%	18.84 %	19.83 %	-1.34%	-9.72%

# Table 1 Development of sectoral stock price indices for the 2013-2019 period

Table 1 shows that the development of sectoral shares fluctuates from year to year. Several sectors experienced positive and negative growth. In 2019, there were 6 groups of shares that experienced depreciation, namely the agriculture sector, mining sector, miscellaneous industrial sector, industrial consumer goods sector, trade services & investment sector, and manufacturing sector. In the consumer goods industrial sector, share price depreciation was quite high, around -20.11% and the highest share price appreciation in 2017 was in the finance sector at 15.22%.



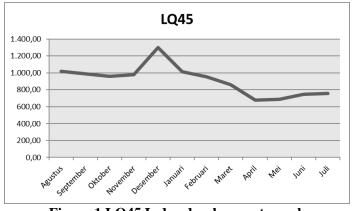


Figure 1 LQ45 Index development graph

The LQ45 index is the market capitalization value of the 45 most liquid stocks and has a large capitalization value, this is an indicator of liquidation. The LQ45 index uses 45 stocks selected based on stock trading liquidity and is adjusted every six months (at the beginning of February and August). Thus, the shares contained in the index will always change. The LQ45 index fluctuates from year to year but in general it experiences growth. This fluctuation occurs as a result of systematic risk and non-systematic risk. Systematic risk is a risk that cannot be avoided. Example of systematic riskare an increase in interest rates (interest rate risk), an increase in inflation (inflationary risk) and high market volatility (market risk). Non-systematic risk is often referred to as specific risk, company risk or un-systematic risk. Non-systematic risk can generally be managed using a portfolio. Examples of non-systematic risks are: liquidity risk, bankruptcy risk (financial / credit risk) and risk of lawsuits (operational risk). There are two models that can be used to form an optimal portfolio, namely the Markowits model and the single index model.

#### **RESEARCH METHODS**

#### 1. Types and Sources of Research Data

The type of data used in this research is descriptive research with quantitative data analysis techniques and is secondary data, namely data that has been published and previously processed. The data collected in this research includes monthly closing stock price data for companies that have gone public on the Indonesia Stock Exchange, monthly IHSG, and interest rates on Bank Indonesia Certificates (SBI). This data was obtained from IDX Statistics for 2017 - 2020, the Indonesian Stock Exchange website.

# 2. Population

The population in this study are all issuers included in the LQ45 calculation for the period February 2018 - July 2020. Based on observations of the population on the LQ45 Index on the Indonesian stock exchange from February 20218 - July 2020, there are 59 shares.

#### 3. Sample

In this research, the author will use a purpsive sampling method by considering the following criteria:

- a) The company is on the LQ 45 Index
- b) The company did not carry out corporate actions during the observation period that directly affected share prices on the IDX.
- c) Companies that are consecutively listed in 5 periods of the LQ 45 index, namely in period
  - February 2018 to July 2018
  - August 2018 to February 2019
  - February 2019 to July 2019
  - August 2019 to February 2020
  - February 2020 to July 2020



#### DISCUSSION Single Index Model Portfolie

# Single Index Model Portfolio Analysis

The Single Index Model is based on the observation that the price of a share fluctuates in the same direction as the market price index. In particular, it can be observed that most shares tend to experience an increase in price if the market share price index rises and vice versa. The calculation of the single index will be explained in seven parts as follows.

#### 1. Return Value & Market Risk

Determining the value of return and market risk in this research is based on the closing price of the LQ 45 Index on the Indonesia Stock Exchange with an observation period from February 2018 to July 2020, in calculating the market index using market realized return (Rm), market expected return E(R(m)) as well as market risk ( $\sigma$ 2) with the market price index at LQ 45 Index as the input variable, the calculation results are in Table 2 as follows.

Market Index	Market Realization	Market Expectations	Market Risk
	(∑Rm)	<i>E</i> ( <b>R</b> (m))	(σ2)
LQ Index 457	-0.269	-0.009	0.003

#### Table 2 Return Market Realization, Market Expectation Return and Market Risk

Based on the research results in Table 4.1 between realized market returns and Market Expectations on the LQ 45 Index for the period February 2018 to July 2020. Meanwhile, regarding the relationship between market expectations and market risk, according to Jogiyanto (2017: 322), return expectations and risk have a positive relationship, the greater the risk of a security or market, the greater the expected return and vice versa, this positive relationship only applies to returns that have not yet occurred. However, in this study this positive relationship did not occur so that the market expectation value in the LQ 45 Index in Table 4.1 shows a low value for market risk, which means that a positive relationship does not always occur.

# 2. Calculating ChangeStock Prices Against the Market and Contrary Price Changes and Calculating Unsystematic Risk

After obtaining the values of return and market risk which have been described in the previous section, the next step is to look for the values of beta, alpha and variance residual error. In this calculation, apart from return and market risk, you also need the value of the return and individual risk of each stock which has also been calculated. described in the results of the optimal portfolio analysis of the Markowitz Model, beta which is the sensitivity of a security's return to returns from the market and beta is used to calculate excess return to beta (ERB) and (Bi), this (Bi) value is also used to calculate (Ci), Beta calculations are carried out using the Excel program with the slope formula, namely calculating the realized return of a stock and calculating the realized return of the market index in a certain period. Alphawhich is the change in market price of a security, is used to find the value (Ai) needed in the calculation (Ci), the alpha value calculation is carried out using the Excel program with the intercept formula, while the residual error variance is used in the calculation to find the value (Ai=Aj), (Bi=Bj) and proportion of funds. Beta, alpha and variance residual error values can be seen in Table 3 below.

No	Code	Issuer	а	β	$\sigma^2$
1	ADRO	Adaro Energy Tbk.	0.138	2,601	0.780
2	AKRA	AKR Corporindo Tbk.	0.019	0.739	0.004
3	ANTM	Aneka Tambang Tbk.	0.019	1,373	0.024
4	ASII	Astra International Tbk.	0.015	1,515	0.023
5	BBCA	Bank Central Asia Tbk.	0.006	0.208	0.004
6	BBNI	Bank Negara Indonesia Tbk.	0.020	2,454	0.045
7	BBRI	Bank Rakyat Indonesia Tbk.	0.009	0.881	0.016

#### Table 3 Value of Share Price Changes to the Market

#### https://jaruda.org



8         BBTN         Bank Tabungan Negara Tbk.         0.013         1,729         0.025           9         BMRI         Bank Mandiri Tbk.         0.007         0.709         0.007           10         BSDE         Bumi Serpong Damai Tbk.         0.009         1,175         0.010           11         EXCL         XL Axiata Tbk.         0.007         2,886         0.059           12         GGRM         Gudang Garam Tbk.         0.006         2,008         0.033           13         HMSP         HM Sampoerna Tbk.         0.005         1,347         0.013           14         ICBP         Tbk.         0.003         1,354         0.029           15         INCO         Vale Indonesia Tbk.         0.001         2,110         0.046           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           17         INTP         Tbk.         -0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.003         1,640         0.023           20         LPPF         Matahari Department Store Tbk.         -0.014         2,381         0.041           22         PGAS						
10         BSDE         Bumi Serpong Damai Tbk.         0.009         1,175         0.010           11         EXCL         XL Axiata Tbk.         0.007         2,886         0.059           12         GGRM         Gudang Garam Tbk.         0.006         2,008         0.033           13         HMSP         HM Sampoerna Tbk.         0.005         1,347         0.013           14         ICBP         Tbk.         0.003         1,354         0.029           15         INCO         Vale Indonesia Tbk.         0.001         2,110         0.046           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           17         INTP         Tbk.         -0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.001         1,707         0.023           20         LPPF         Matahari Department Store Tbk.         -0.003         1,640         0.0023           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         -0.004         0.956         0.009           24 <td< td=""><td>8</td><td>BBTN</td><td>Bank Tabungan Negara Tbk.</td><td>0.013</td><td>1,729</td><td>0.025</td></td<>	8	BBTN	Bank Tabungan Negara Tbk.	0.013	1,729	0.025
11         EXCL         XL Axiata Tbk.         0.007         2,886         0.059           12         GGRM         Gudang Garam Tbk.         0.006         2,008         0.033           13         HMSP         HM Sampoerna Tbk.         0.005         1,347         0.013           14         ICBP         Tbk.         0.003         1,354         0.029           15         INCO         Vale Indonesia Tbk.         0.001         2,110         0.046           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           16         INDF         Indocement Tunggal Prakarsa         -0.001         1,707         0.024           17         INTP         Tbk.         -0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.001         1,707         0.023           20         LPPF         Matahari Department Store Tbk.         -0.003         1,640         0.023           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         -0.004         0.956         0.009           23	9	BMRI	Bank Mandiri Tbk.	0.007	0.709	0.007
12         GGRM         Gudang Garam Tbk.         0.006         2,008         0.033           13         HMSP         HM Sampoerna Tbk.         0.005         1,347         0.013           14         ICBP         Tbk.         0.003         1,354         0.029           15         INCO         Vale Indonesia Tbk.         0.001         2,110         0.046           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.024           17         INTP         Tbk.         -0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.001         1,275         0.013           19         KLBF         Kalbe Farma Tbk.         -0.003         1,640         0.023           20         LPPF         Matahari Department Store Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         0.011         0.466         0.006           23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP<	10	BSDE	Bumi Serpong Damai Tbk.	0.009	1,175	0.010
13         HMSP         HM Sampoerna Tbk.         0.005         1,347         0.013           14         ICBP         Tbk.         0.003         1,354         0.029           15         INCO         Vale Indonesia Tbk.         0.001         2,110         0.046           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           16         INDF         Indocement Tunggal Prakarsa         -         -         -           17         INTP         Tbk.         -0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.001         1,707         0.023           19         KLBF         Kalbe Farma Tbk.         -0.003         1,640         0.023           20         LPPF         Matahari Department Store Tbk.         -0.003         1,093         0.015           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         0.001         0.466         0.006           23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP <td>11</td> <td>EXCL</td> <td>XL Axiata Tbk.</td> <td>0.007</td> <td>2,886</td> <td>0.059</td>	11	EXCL	XL Axiata Tbk.	0.007	2,886	0.059
Indofood CBP Sukses Makmur         0.003         1,354         0.029           14         ICBP         Tbk.         0.003         1,354         0.029           15         INCO         Vale Indonesia Tbk.         0.001         2,110         0.046           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           Indocement Tunggal Prakarsa         -         -         -         0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.001         1,707         0.023           19         KLBF         Kalbe Farma Tbk.         -0.003         1,640         0.023           20         LPPF         Matahari Department Store Tbk.         -0.003         1,093         0.015           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.008         0.999         0.013           25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SM	12	GGRM	Gudang Garam Tbk.	0.006	2,008	0.033
14ICBPTbk.0.0031,3540.02915INCOVale Indonesia Tbk.0.0012,1100.04616INDFIndofood Sukses Makmur Tbk.0,0001,7790.02316INDFIndocement Tunggal Prakarsa-0.0011,7070.02417INTPTbk0.0011,7070.02418JSMRJasa Marga Tbk0.0011,2750.01319KLBFKalbe Farma Tbk0.0031,6400.02320LPPFMatahari Department Store Tbk0.0031,0930.01521MNCNMedia Nusantara Citra Tbk0.0142,3810.04122PGASPerusahaan Gas Negara Tbk.0.0010.4660.00623PTBABukit Asam Tbk0.0080.9990.01325SCMASurya Citra Media Tbk0.0151.2560.01526SMGRSemen Indonesia Tbk0.0130.8730.01527SRILSri Rejeki Isman Tbk0.0351,7160.03928TLKMTelekomunikasi Indonesia Tbk0.0020.9440.01130WIKAWijaya Karya Tbk0.0110.4640.009	13	HMSP	HM Sampoerna Tbk.	0.005	1,347	0.013
15         INCO         Vale Indonesia Tbk.         0.001         2,110         0.046           16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           Indocement Tunggal Prakarsa         -0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.001         1,707         0.023           19         KLBF         Kalbe Farma Tbk.         -0.001         1,275         0.013           20         LPPF         Matahari Department Store Tbk.         -0.003         1,640         0.023           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         0.001         0.466         0.006           23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.008         0.999         0.013           25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.035         1,716         0.039           28         TLKM			Indofood CBP Sukses Makmur			
16         INDF         Indofood Sukses Makmur Tbk.         0,000         1,779         0.023           17         INTP         Tbk.         -0.001         1,707         0.024           18         JSMR         Jasa Marga Tbk.         -0.001         1,275         0.013           19         KLBF         Kalbe Farma Tbk.         -0.003         1,640         0.023           20         LPPF         Matahari Department Store Tbk.         -0.003         1,093         0.015           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         0.001         0.466         0.006           23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.035         1,716         0.039           28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           2	14	ICBP	Tbk.	0.003	1,354	0.029
Indocement Tunggal Prakarsa-0.0011,7070.02417INTPTbk0.0011,7070.02418JSMRJasa Marga Tbk0.0011,2750.01319KLBFKalbe Farma Tbk0.0031,6400.02320LPPFMatahari Department Store Tbk0.0031,0930.01521MNCNMedia Nusantara Citra Tbk0.0142,3810.04122PGASPerusahaan Gas Negara Tbk.0.0010.4660.00623PTBABukit Asam Tbk0.0040.9560.00924PTPPPP Tbk0.0080.9990.01325SCMASurya Citra Media Tbk0.0151.2560.01526SMGRSemen Indonesia Tbk0.0351,7160.03928TLKMTelekomunikasi Indonesia Tbk0.0040.3430.00329UNTRUnited Tractors Tbk0.0110.4640.009	15	INCO	Vale Indonesia Tbk.	0.001	2,110	0.046
17INTPTbk0.0011,7070.02418JSMRJasa Marga Tbk0.0011,2750.01319KLBFKalbe Farma Tbk0.0031,6400.02320LPPFMatahari Department Store Tbk0.0031,0930.01521MNCNMedia Nusantara Citra Tbk0.0142,3810.04122PGASPerusahaan Gas Negara Tbk.0.0010.4660.00623PTBABukit Asam Tbk0.0040.9560.00924PTPPPP Tbk0.0080.9990.01325SCMASurya Citra Media Tbk0.0130.8730.01526SMGRSemen Indonesia Tbk0.0351,7160.03928TLKMTelekomunikasi Indonesia Tbk0.0040.3430.00329UNTRUnited Tractors Tbk0.0220.9440.01130WIKAWijaya Karya Tbk0.0110.4640.009	16	INDF	Indofood Sukses Makmur Tbk.	0,000	1,779	0.023
18         JSMR         Jasa Marga Tbk.         -0.001         1,275         0.013           19         KLBF         Kalbe Farma Tbk.         -0.003         1,640         0.023           20         LPPF         Matahari Department Store Tbk.         -0.003         1,093         0.015           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         0.001         0.466         0.006           23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.008         0.999         0.013           25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009			Indocement Tunggal Prakarsa			
19         KLBF         Kalbe Farma Tbk.         -0.003         1,640         0.023           20         LPPF         Matahari Department Store Tbk.         -0.003         1,093         0.015           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         0.001         0.466         0.006           23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.008         0.999         0.013           25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.035         1,716         0.039           28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.011         0.464         0.009	17	INTP	Tbk.	-0.001	1,707	0.024
20         LPPF         Matahari Department Store Tbk.         -0.003         1,093         0.015           21         MNCN         Media Nusantara Citra Tbk.         -0.014         2,381         0.041           22         PGAS         Perusahaan Gas Negara Tbk.         0.001         0.466         0.006           23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.008         0.999         0.013           25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.035         1,716         0.039           28         TLKM         Telekomunikasi Indonesia Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	18	JSMR	Jasa Marga Tbk.	-0.001	1,275	0.013
21MNCNMedia Nusantara Citra Tbk0.0142,3810.04122PGASPerusahaan Gas Negara Tbk.0.0010.4660.00623PTBABukit Asam Tbk0.0040.9560.00924PTPPPP Tbk0.0080.9990.01325SCMASurya Citra Media Tbk0.0151.2560.01526SMGRSemen Indonesia Tbk0.0130.8730.01527SRILSri Rejeki Isman Tbk0.0351,7160.03928TLKMTelekomunikasi Indonesia Tbk0.0040.3430.00329UNTRUnited Tractors Tbk0.0110.4640.009	19	KLBF	Kalbe Farma Tbk.	-0.003	1,640	0.023
21MNCNMedia Nusantara Citra Tbk0.0142,3810.04122PGASPerusahaan Gas Negara Tbk.0.0010.4660.00623PTBABukit Asam Tbk0.0040.9560.00924PTPPPP Tbk0.0080.9990.01325SCMASurya Citra Media Tbk0.0151.2560.01526SMGRSemen Indonesia Tbk0.0130.8730.01527SRILSri Rejeki Isman Tbk0.0351,7160.03928TLKMTelekomunikasi Indonesia Tbk0.0040.3430.00329UNTRUnited Tractors Tbk0.0110.4640.009	20	LPPF	Matahari Department Store Tbk.	-0.003	1,093	0.015
23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.008         0.999         0.013           25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.035         1,716         0.039           28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	21	MNCN		-0.014	2,381	0.041
23         PTBA         Bukit Asam Tbk.         -0.004         0.956         0.009           24         PTPP         PP Tbk.         -0.008         0.999         0.013           25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.035         1,716         0.039           28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	22	PGAS	Perusahaan Gas Negara Tbk.	0.001	0.466	0.006
25         SCMA         Surya Citra Media Tbk.         -0.015         1.256         0.015           26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.035         1,716         0.039           28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	23	PTBA		-0.004	0.956	0.009
26         SMGR         Semen Indonesia Tbk.         -0.013         0.873         0.015           27         SRIL         Sri Rejeki Isman Tbk.         -0.035         1,716         0.039           28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	24	PTPP	PP Tbk.	-0.008	0.999	0.013
27SRILSri Rejeki Isman Tbk0.0351,7160.03928TLKMTelekomunikasi Indonesia Tbk0.0040.3430.00329UNTRUnited Tractors Tbk0.0220.9440.01130WIKAWijaya Karya Tbk0.0110.4640.009	25	SCMA	Surya Citra Media Tbk.	-0.015	1.256	0.015
28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	26	SMGR	Semen Indonesia Tbk.	-0.013	0.873	0.015
28         TLKM         Telekomunikasi Indonesia Tbk.         -0.004         0.343         0.003           29         UNTR         United Tractors Tbk.         -0.022         0.944         0.011           30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	27	SRIL	Sri Rejeki Isman Tbk.	-0.035	1,716	0.039
30         WIKA         Wijaya Karya Tbk.         -0.011         0.464         0.009	28	TLKM		-0.004		0.003
	29	UNTR	United Tractors Tbk.	-0.022	0.944	0.011
	30	WIKA	Wijaya Karya Tbk.	-0.011	0.464	0.009
	31	WSKT	Waskita Karya Tbk.	-0.010	0.286	0.008

The research results in Table 3 show that the alpha value ( $\alpha$ ) in Table 4.2 ranges from -0.0349 to 0.1383. If the alpha value is positive, it indicates that the stock performance is better compared to the market index. The higher it is of course the better, if the alpha value negative means that the stock performance is worse compared to the market index and the lower it is, the worse it is. beta ( $\beta$ ) for each stock ranges from 0.2081 to 2.2886. If the stock has a beta value equal to or more than -0.009 then the stock's sensitivity is the same as the market sensitivity. In other words, if the market index falls by -0.009 or 1% then the stock also fell by 1%. In the context of this research, the highest beta value was obtained by PP (Persero) Tbk (PTPP) of 2,Risk ( $\sigma$ 2) in Table 4.2 is the variance of the residual error of the stock (i) which is also a unique or unsystematic risk. The greater the value of variance ei, the greater the unsystematic risk that will be borne by investors and vice versa. Unsystematic risk is a risk that can be eliminated by diversifying, factors related to this risk such as capital structure, asset structure, level of liquidity, level of profit and so on.

#### 3. ReturnRisk-Free Assets

AssetsRisk-free is an asset that has a certain expected return with a risk equal to zero (Jogiyanto: 2008: 301), meaning that if an investor invests his funds in a type of asset, he can know the amount of profit that will be obtained in the future. In this research, risk-free asset returns are sourced from Bank Indonesia for the period February 2018 to July 2020, the average monthly risk-free asset return value will be used as the value (Rf) for calculations for optimal portfolio formation, as for the calculation of risk-free asset returns in Table 4 is as follows.

Mandh	YEAR			
Month	2018	2019	2020	
Jan		6.00%	5.00%	
Feb	4.25%	6.00%	4.75%	

### Table 4 Return Risk-Free Assets



Mar	4.25%	6.00%	4.50%
Apr	4.25%	6.00%	4.50%
May	4.75%	6.00%	4.50%
Jun	5.25%	6.00%	4.25%
Jul	5.25%	5.75%	4.00%
Aug	5.50%	5.50%	
Sept	5.75%	5.25%	
Oct	5.75%	5.00%	
Nov	6.00%	5.00%	
Dec	6.00%	5.00%	
Average/year	5.18%	5.63%	4.50 %
Average/month (Rf)	5.17%		

Based on the research results in Table 4.4, the average growth in risk-free assets from 2018 to 2020 increased with a decrease of 0.45%.

Meanwhile, for the research results in Table 4.3, the average growth in risk-free assets from 2019 to 2020 decreased by a difference of 1.16%.

#### 4. Optimal Portfolio Forming Stocks

The shares selected in the optimal portfolio are shares that have a high excess return to beta (ERB) value, shares with a low excess return to beta (ERB) value will not be included in the optimal portfolio, so a limiting point is needed that determines the excess return value high to beta (ERB), which is meant by excess return to beta (ERB) is defined as the difference between the expected return and the return on risk-free assets, the cut off point (C\*) is obtained from the largest value (Ci), which is meant by (Ci) is the value (C) for security I which is calculated by accumulating the values (A1) to (Ai) and also the values (B1) to (Bi), the shares forming the optimal portfolio are those that have a value (ERB) of more is large or equal to the limiting point value (C\*), Shares whose value (ERB) is smaller than the limiting point (C\*) will not be included in the optimal portfolio, while the shares selected in the optimal portfolio are in Table 4.5 as follows.

No	Share	ERB	Ci	C*	Note
1	ADRO	-0.0283	-0.0036	0.0039	Not optimal
2	AKRA	-0.0136	-0.0048	0.0039	Not optimal
3	ANTM	0.0426	0.0011	0.0039	Optimal
4	ASII	-0.0129	-0.0048	0.0039	Not optimal
5	BBCA	0.0103	0.0039	0.0039	Optimal
6	BBNI	-0.0112	-0.0047	0.0039	Not optimal
7	BBRI	-0.0053	-0.0022	0.0039	Not optimal
8	BBTN	-0.0107	-0.0031	0.0039	Not optimal
9	BMRI	-0.0084	-0.0036	0.0039	Not optimal
10	BSDE	-0.0243	-0.0075	0.0039	Not optimal
11	EXCL	-0.0038	-0.0005	0.0039	Not optimal
12	GGRM	-0.0175	-0.0051	0.0039	Not optimal
13	HMSP	-0.0369	-0.0092	0.0039	Not optimal
14	ICBP	-0.0023	-0.0001	0.0039	Not optimal
15	INCO	0.0015	0.0003	0.0039	Not optimal
16	INDF	-0.0168	-0.0019	0.0039	Not optimal
17	INTP	-0.0156	-0.0037	0.0039	Not optimal
18	JSMR	-0.0042	-0.0015	0.0039	Not optimal
19	KLBF	-0.0050	-0.0010	0.0039	Not optimal

#### Table 5 Formation of an Optimal Portfolio with a Single Index Model

ANALYSIS OF OPTIMAL STOCK PORTFOLIO INVESTMENT IN LQ45 INDEX USES THE MARKOWITZ MODEL AND SINGLE INDEX MODEL Fadlan Alkindi<sup>1</sup>, Isfenti Sadalia<sup>2</sup>, Iskandar Muda<sup>3</sup>

#### https://jaruda.org



20	LPPF	-0.0318	-0.0072	0.0039	Not optimal
21	MNCN	-0.0099	-0.0019	0.0039	Not optimal
22	PGAS	-0.0084	-0.0030	0.0039	Not optimal
23	PTBA	-0.0602	-0.0018	0.0039	Not optimal
24	PTPP	-0.0080	-0.0034	0.0039	Not optimal
25	SCMA	-0.0121	-0.0044	0.0039	Not optimal
26	SMGR	-0.0019	-0.0006	0.0039	Not optimal
27	SRIL	-0.0215	-0.0050	0.0039	Not optimal
28	TLKM	-0.0342	-0.0036	0.0039	Not optimal
29	UNTR	-0.0429	-0.0029	0.0039	Not optimal
30	WIKA	-0.0027	-0.0011	0.0039	Not optimal
31	WSKT	-0.0166	-0.0068	0.0039	Not optimal

Based on the research results in Table 5, Aneka Tambang Tbk obtained the highest (ERB) value of 0.0426The highest (Ci) value obtained by Bank Central Asia Tbk was 0.0039 which will be used as the limiting point (C\*) because it has the highest (Ci) value. The criteria for determining the inclusion of a security in the optimal portfolio is if the value (ERB) of a share is equal to or greater than the limiting point value (C\*) so that the shares included in the optimal portfolio are Aneka Tambang Tbk and Bank Central Asia Tbk. Shares that are included in the optimal portfolio have a positive value, which means that each individual's expected return is greater than the return on risk-free assets. This illustrates that shares that are included in the optimal portfolio of the Single Index Model are companies that have a high level of return. higher than investing in risk-free securities or assets, namely Bank Indonesia Certification (SBI), excess return to beta (ERB) is used to measure stock premium returns relative to a unit of risk that cannot be diversified which is measured by beta. (ERB) shows the relationship between return and risk which is a determining factor in investment.

# 5. Proportion of Funds in Securities for Optimal Portfolio Formation

After determining the selected securities in forming an optimal portfolio as in Table 4.4, the next step is to determine the proportion of funds that will be invested by investors in each selected security in the optimal portfolio using a single index model. The proportion of funds for each security that will be invested can be seen in Table 4.6 as following.

No	Share	Code	Xi	Wi
1	Aneka Tambang Tbk	ANTM	0.129	10.5%
2	2 Bank Central Asia Tbk BBCA			89.5%
	Amount	1.129	100%	

# Table 6 Calculation of the Proportion of a Single Index Model Stock Fund

# 6. Calculating the Value of Changes in Securities Portfolio Prices Against the Market and the Value of Changes in Portfolio Prices Otherwise

Before calculating the return and risk of a single index model portfolio, the beta and alpha values of the portfolio must first be known. The beta and alpha of the portfolio are input in calculating the return and risk of the portfolio. The beta value is also called the sensitivity of changes in securities to the market, while the beta of the portfolio is the beta of the individual shares. included in the formation of an optimal portfolio with the proportion of funds invested, while the portfolio alpha is the individual alpha of the securities selected in the optimal portfolio with the proportion of funds invested, the Beta and Alpha values of the portfolio in Table 7 are as follows.

 Table 7 Value of Changes in Portfolio Prices to the Market

No	Share	Code	ар	βp
1	Aneka Tambang Tbk	ANTM	0.1383	0.2729
2	Bank Central Asia Tbk	BBCA	0.0186	0.6611

650



Based on the research results in Table 4.6, the highest portfolio alpha value ( $\alpha \rho$ ) is Aneka Tambang, Tbk, while the highest portfolio beta value ( $\beta \rho$ ) is obtained by Bank Central Asia, Tbk. While the portfolio alpha and beta are used in calculating portfolio return and risk, the portfolio alpha and beta are influenced by the proportion of funds for each security, the greater the proportion of funds in a security, the greater the beta and alpha values of the security portfolio and vice versa.

#### 7. Returnand Single Index Model Portfolio Risk

Calculating portfolio return and risk in the single index model can be used to measure return expectations and portfolio variance, in calculating portfolio return and risk in the previous Markowitz model using the covariance value of each combination of two securities, whereas in the single index model it only requires the beta and alpha values of the portfolio, as for Portfolio returns and risks using the Single Index Model can be seen in Table 8 as follows.

No	Share	Code	Return E(Rp)	Risk $\sigma^2$
1	Aneka Tambang Tbk	ANTM	0.1359	0.0821
2	Bank Central Asia Tbk	BBCA	0.0127	0.0052
	Amount	0.1486	0.0873	

# Table 8 Returnand Single Index Model Risk

Based on the research results in Table 4.7, the portfolio return value  $(ER\rho)$  of the Single Index Model is 0.1486 while the portfolio risk  $(\sigma\rho 2)$  is 0.0873, this means that forming an optimal portfolio using the Single Index Model is able to produce a portfolio return of 14.8% with 2 different types of issuers if investors invest their funds at a risk level of 8.7%.

#### Markowitz Model Portfolio Analysis

Determining the value of the return and risk of each stock using the Markowitz model can be used to measure realized return, expected return and standard deviation for each security. Realized return is used to measure actual profits based on historical data on stock prices in the past while expected return measures the level of profit that obtained in the future using historical data on stock prices in the past. To measure the risk borne by investors in the future, standard deviation is used using historical data on realized returns and expected return data for each stock. The data processed is monthly closing stock price data for the period February 2018 to July 2020, as for expected returns. and the risks of each stock in the research sample.

#### Table 9 ReturnExpectations and Risks for Each Stock in the LQ 45 Index

No	Code	Issuer	E(Ri)	σ2
1	ADRO	Adaro Energy Tbk.	-0.0204	0.0132
2	AKRA	AKR Corporindo Tbk.	-0.0179	0.0146
3	ANTM	Aneka Tambang Tbk.	0.1150	0.7600
4	ASII	Astra International Tbk.	-0.0122	0.0083
5	BBCA	Bank Central Asia Tbk.	0.0120	0.0027
6	BBNI	Bank Negara Indonesia Tbk.	-0.0155	0.0132
7	BBRI	Bank Rakyat Indonesia Tbk.	-0.0019	0.0061
8	BBTN	Bank Tabungan Negara Tbk.	-0.0182	0.0329
9	BMRI	Bank Mandiri Tbk.	-0.0070	0.0073
10	BSDE	Bumi Serpong Damai Tbk.	-0.0262	0.0106
11	EXCL	XL Axiata Tbk.	0.0010	0.0137
12	GGRM	Gudang Garam Tbk.	-0.0124	0.0067
13	HMSP	HM Sampoerna Tbk.	-0.0305	0.0081

#### https://jaruda.org



-			
14	ICBP	Indofood CBP Sukses Makmur Tbk.	0.0038 0.0041
15	INCO	Vale Indonesia Tbk.	0.0063 0.0187
16	INDF	Indofood Sukses Makmur Tbk.	-0.0035 0.0052
17	INTP	Indocement Tunggal Prakarsa Tbk.	-0.0127 0.0115
18	JSMR	Jasa Marga Tbk.	-0.0029 0.0163
19	KLBF	Kalbe Farma Tbk.	0.0008 0.0058
20	LPPF	Matahari Department Store Tbk.	-0.0503 0.0299
21	MNCN	Media Nusantara Citra Tbk.	-0.0091 0.0232
22	PGAS	Perusahaan Gas Negara Tbk.	-0.0125 0.0214
23	PTBA	Bukit Asam Tbk.	-0.0129 0.0080
24	PTPP	PP Tbk.	-0.0188 0.0343
25	SCMA	Surya Citra Media Tbk.	-0.0164 0.0151
26	SMGR	Semen Indonesia Tbk.	0.0015 0.0160
27	SRIL	Sri Rejeki Isman Tbk.	-0.0171 0.0098
28	TLKM	Telekomunikasi Indonesia Tbk.	-0.0074 0.0030
29	UNTR	United Tractors Tbk.	-0.0156 0.0087
30		Wijaya Karya Tbk.	-0.0022 0.0269
31	WSKT	Waskita Karya Tbk.	-0.0351 0.0240

Based on the relationship between expected return and risk according to Jogiyanto (2017:322), the greater the risk of a security, the greater the expected return, the opposite is also true, namely the smaller the expected return, the smaller the risk borne, this positive relationship only applies to expected return or ex -ante return (before the fact) is for returns that have not yet occurred, however in Figure 4.1 the positive relationship only occurs for BBCA shares, which means that the relationship between return and risk does not always show a positive relationship. According to Jogiyanto (2017:388) the optimal portfolio of the Markowitz model is in the efficient set, which means that shares must show a positive expected return value, so that in this research shares that have a negative or zero expected return value will not be included in the next calculation, which means they will not provide profits. during the period February 2018 to July 2020, it was determined that those included in the optimal portfolio calculation were 7 issuers as follows.

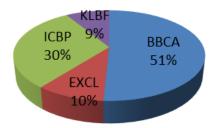
**Table 10 Optimal Portfolio Formation Stocks** 

No	Code	Issuer	E(Ri)	σ2
1	ANTM	Aneka Tambang Tbk.	0.115	0.76
2	BBCA	Bank Central Asia Tbk.	0.012	0.003
3	EXCL	XL Axiata Tbk.	0.001	0.014
4	ICBP	Indofood CBP Sukses Makmur Tbk.	0.0038	0.004
5	INCO	Vale Indonesia Tbk.	0.0063	0.019
6	KLBF	Kalbe Farma Tbk.	0.0008	0.006
7	SMGR	Semen Indonesia Tbk.	0.0015	0.016

#### **Optimal Portfolio Markowitz Model**

Optimal portfolio The Markowitz model assumes that an optimal portfolio is in the efficient set (Jogiyanto 2017:388). An efficient portfolio is a portfolio that provides the greatest level of profit with the same risk or the smallest risk with the same level of profit (Husnan 2015:71), with By forming a portfolio, a combination can be obtained that dominates certain shares. Based on the results of research using linear solving, an optimal portfolio is formed with a relationship between portfolio return and portfolio risk with the proportions as shown in the following figure:





# Figure 2 Markowitz Model Optimal Portfolio Fund Proportions

Based on the results of the linear solver, the optimal portfolio for the Markowitz Model stock portfolio is only four stocks with a Portfolio Return of 0.0074 and a Portfolio Risk of 0.428 with the largest composition being 50.81% owned by Bank Central Asia Tbk with individual risk.0.003as well as expected returns0.012 and the smallest composition of 8.77% is owned by Kalbe Farma Tbk. With an individual risk of 0.006 and an expected return of 0.0008.

#### DISCUSSION

The LQ 45 index is the market capitalization value of the 45 most liquid shares and has a large capitalization value, this is an indicator of liquidation. The LQ45 Index uses 45 stocks selected based on stock trading liquidity and is adjusted every six months. In this study, the LQ 45 Index's movements were observed from February 2018 to July 2020 with a total population of 59 issuers and 31 issuers as candidate samples. Based on the results of research using the Markowitz model, an optimal portfolio was obtained with a portfolio return of 0.0074, while the portfolio risk was 0.428 and the proportion of funds formed was BBCA 50.81%, EXCL 9.83%, ICBP 30.59%, and KLBF 8. 77% for the Single Index model research results obtained two shares forming an optimal portfolio with a portfolio return of 0.1486 and a risk of 0.0873 while the proportion of funds formed by ANTM was 10.5%, and BBCA 89.5%, as for the comparison of model research results Markowitz and the Single Index Model as follows:

No	Information	Markowitz mod-	Single Index
110	mormation	el	Model
1	Selected Shares and proportion	1. BBCA	
	of funds	50.81%	1. ANTM 10.5%
		2. EXCL 9.83%	2. BBCA 89.5%
		3. ICBP 30.59%	
		4. KLBF 8.77%	
2	Optimal Portfolio Return	0.0074	0.1486
3	Optimal Portfolio Risk	0.0428	0.0873

 Table 11 Comparison Table of Results of the Markowitz Model and Single Index Model

Based on the comparison of the expected return of the Markowitz model and the single index model in Table 4.10, the expected return of the Markowitz model is 0.0074 and the portfolio risk is 0.0428, while the expected return of the Single Index model is 0.1486 and the portfolio risk is 0.0873, these results show that portfolios using a single index model in analyzing optimal portfolios produce higher returns than the Markowitz model with a greater risk value. The greater proportion of shares in the Markowitz model reduces the greater level of risk.

# CONCLUSION

Based on the results of the optimal portfolio analysis of the Markowitz model and Single Index Model, it can be concluded as follows:

- 1. Based on the optimal portfolio formation of the Markowitz Model, 4 shares form a portfolio return expectation of 0.0074, while the portfolio risk is 0.0428 and the proportion of funds formed is BBCA 50.81%, EXCL 9.83%, ICBP 30.59%, and KLBF 8.77%
- 2. Based on the formation of a single index model portfolio, 2 optimal portfolio formations were obtained



with a portfolio return of 0.1486 and a risk of 0.0873, while the proportion of funds formed by ANTM was 10.5%, and BBCA was 89.5%.

3. Based on research results, it proves that the single index model can produce a profit of 14.86% with a risk level of 8.73% compared to the Risk-Free Asset Return Rate of 5.17%. Meanwhile, the Markowitz model can produce a portfolio return of 0.74% with a portfolio risk of 4.28%, which does not provide optimal profits because the expected return from the Markowitz model portfolio is lower than the Risk-Free Asset Return Rate.

#### REFERENCES

Agus S. (2013).VAR Portfolio Optimal: Perbandingan Antara Markowitz dan Mean Absolute Deviation; p.37-50. Bodie, Z., Kane, A., & Marcus, A.J. (2008).Essentials of investment (7thed.). London: McGraw-Hill.

- Durianto, Darmadi, Sugiarto, dan Toni Sitinjak. (2001). Strategi Menaklukan Pasar : Melalui Riset Ekuitas dan Perilaku Merek. Jakarta: PT Gramedia Pustaka Utama.
- Eko, Umanto. (2008). Analisis dan Penilaian Kinerja Portofolio Optimal Saham-Saham LQ-45.Bisnis dan Birokrasi, Jurnal Ilmu Administrasi dan Organisasi, FISIP Universitas Indonesia; p.178-187.
- Francis, Jack. (1991). Investment : Analysis and management (5th ed.). Singapore : Mc.Graw Hill Inc; p.1.
- Jogiyanto. 2017. Teori Portofolio dan Analisis Investasi. Edisi Kelima. BPFE. Yogyakarta
- K.Reilly, Frank; C.Brown, Keith. (2003). Investment Analysis & Portfolio Management (7thed.). Thomson Learning; p.211.
- K.Reilly, Frank; C.Brown, Keith.(2003). Investment Analysis & Portfolio Management (7thed.).US : Thomson South-Western Inc; p.211.
- Mandal, Niranjan. (2013). Sharpe's Single Index Model and Its Application to Construct Optimal Portfolio : An Empirical Study. Great Lakes Herald, Vol.7, No.1.
- Markowitz, Harry. (1952). Portfolio Selection. The Journal of Finance, Vol.7, No.1; pp. 77-91.

Marsalina, Finanda. (2009). Identifikasi Kemampuan Selectivity dan Market Timing Pada Reksa Dana Saham di Indonesia.

- Nancy, M.Szigethy. (2012). Investment Bulletin for Endowment and Foundation Community. The NMS Exchange. Ni Wayan, Y.I ; Ni Putu, A.D. (2013). Penentuan Portofolio Optimal dengan Model Markowitz pada Saham Perbankan di Bursa Efek Indonesia
- Priyo.P, Eko; Nugraha, Ubaidillah; Arlina, Ariefa (2001). Reksa dana: Solusi Perencanaan Investasi di Era Modern. Capital Investment, p.300. Gramedia Pustaka Utama. Savabi, Farhad ; Hamid, S. ; and Bijan, B. (2012).
- Generalization and Combination of Markowitz Sharpe's Theories and New Efficient Frontier Algorithm. African Journal of Business Management Vol. 6(18); pp.5844-5851.
- Suqaier, Faten Shukri, Hussein Ali Al Ziyud. (2011). The Effect of Diversification on Achieving Optimal Portfolio.European Journal of Economics Finance, and Administration Sciences, (32).

Tandelilin, Eduardus. (2010). Portofolio dan Investasi. Teori dan Aplikasi. Edisi Pertama. Yogyakarta: Kanisius.